A Boundary Condition to Absorb Both Propagating and Evanescent Waves in a Finite-Difference Time-Domain Simulation

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***Abstract*—A boundary condition that absorbs both propagating and evanescent waves in a finite-difference time-domain (FDTD) simulation is presented. A computationally efficient method of determining the values of the boundary condition input parameters is described. To illustrate the effectiveness of this boundary condition, it is used to truncate the computational domain of two FDTD simulations, namely a uniform microstrip line and a microstrip line with a gap discontinuity. In both cases, the results are found to be more accurate than those obtained with the conventional second-order Mur absorbing boundary condition.**

I. Introduction

The finite-difference time-domain (FDTD) method is frequently used to model digital and microwave integrated circuits, which predominantly consist of guided wave structures, such as microstrips. Typically, these are open region problems requiring absorbing boundary conditions (ABC's) to truncate the computational space.

Two different types of absorbing boundaries are needed to model open, guided-wave structures, such as microstrips. The walls terminating the computational domain in the longitudinal direction must absorb normally incident waves with a fairly wide range of propagation velocities. Boundary conditions suitable for this purpose have been described in [1], and the reader is referred to this communication for additional detail.

Although the fields impinging upon the side walls used to truncate the computational domain in the transverse direction are primarily evanescent, they have, nonetheless, some contribution from surface waves that propagate outwards. Furthermore, many problems of interest involve some type of discontinuity in the microstrip, and the fields scattered by the discontinuity will, in general, contain radiated waves that propagate toward the side walls. It is essential, therefore, that the boundary condition enforced on the side walls absorb both traveling and evanescent waves.

Typically, the characterization of microstrip via FDTD is carried out by terminating the side walls with ABC's which absorb only traveling waves. Such simulations can produce acceptable results only if the side walls are sufficiently far away from the microstrip conductor that evanescent modes are negligible near them. To meet this requirement it is often necessary to use a large computational domain, which greatly increases both computer storage and run time requirements. This communication presents an ABC that absorbs both evanescent and traveling waves, and thus, allows accurate and efficient microstrip simulations on a smaller computational domain than that required when a purely traveling wave ABC is employed.

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II. Combined Evanescent and Traveling Wave ABC

Consider a boundary on the $x = x_{\max}$ plane. The boundary condition

$$\left(\frac{\partial}{\partial x} + \frac{1}{\nu}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x} + \alpha\right)E = 0 \tag{1}$$

is designed to annihilate both traveling and exponentially attenuating waves impinging upon a boundary. The first factor absorbs waves of the form $f(x - \nu t)$, while the second factor absorbs fields of the form $f(t)e^{-\alpha x}$, with arbitrary y and z dependence. Discretizing this ABC on the $x = M\Delta x$ boundary via central differences yields

$$\begin{aligned}E_M^n = &\left(\frac{1}{2\nu\Delta t + 2\Delta x + \alpha\nu\Delta t\Delta x}\right)\\ &\cdot\left[(-2\nu\Delta t + 2\Delta x - \alpha\nu\Delta t\Delta x)E_M^{n-1}\right.\\ &+ \left(4\nu\Delta t - 4\alpha\Delta x^2\right)E_{M-1}^n\\ &+ \left(4\nu\Delta t + 4\alpha\Delta x^2\right)E_{M-1}^{n-1}\\ &+ (-2\nu\Delta t + 2\Delta x + \alpha\nu\Delta t\Delta x)E_{M-2}^n\\ &\left.+ (-2\nu\Delta t - 2\Delta x + \alpha\nu\Delta t\Delta x)E_{M-2}^{n-1}\right],\end{aligned} \tag{2}$$

where subscripts and superscripts denote the x index and time step, respectively, and the values of the ν and α parameters are yet to be chosen.

This ABC is not very sensitive to ν, and it can be set equal to the speed of light in whatever medium lies next to the boundary. At the boundaries intersected by the microstrip dielectric-air interface, ν is set equal to c_o and $c_{\text{dielectric}}$ on the portions of the boundaries adjacent to air and dielectric, respectively.

The determination of α is more involved. We use a finite-difference approach to solve the Laplace equation for the static field distribution of the microstrip. The computational domain is made somewhat larger than the corresponding cross-section used in the subsequent FDTD simulation. We assume the fields behave as $\exp(-\alpha x)$ in the region of $x = M\Delta x$; hence,

$$\alpha_{x=M\Delta x} = \left.\frac{-1}{E}\frac{\partial E}{\partial x}\right|_{x=M\Delta x}$$

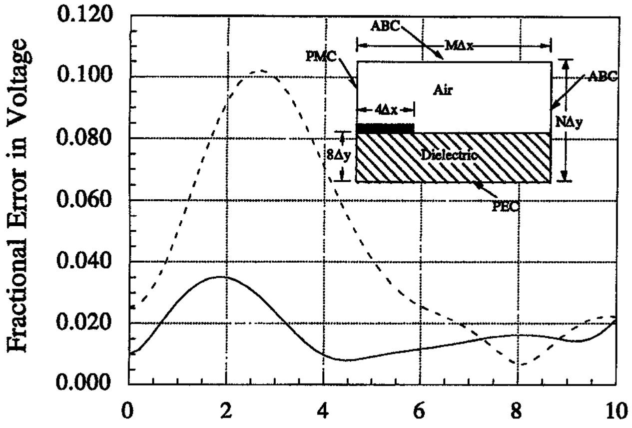

Fig. 1. Errors caused by side ABC's in the FDTD analysis of a uniform microstrip line. Solid line: the evanescent ABC of this letter with α at $M\Delta x = 281$ and α at $N\Delta y = 237$. Dashed line: second-order Mur ABC. Inset shows the cross-section of the microstrip modeled; in this case, the cross-section is $24\Delta x$ by $24\Delta y$, and the cell size in all directions is 0.3175 mm.

$$= \frac{1}{N}\sum_{j=1}^{N}\frac{E(M-1,j)-E(M+1,j)}{2\Delta x E(M,j)}, \quad (3)$$

where E is the magnitude of the E field; the numbers in brackets indicate the x and y indices of the Laplace field nodes, which are assumed to coincide with those of the FDTD simulation; and the side boundaries of the FDTD simulation are at $x = M\Delta x$ and $y = N\Delta y$. The finite-difference solution of the two-dimensional Laplace equation takes a negligible amount of time compared to the solution of the three-dimensional FDTD problem, so the determination of α involves little computational overhead. Although the value of α thus determined is really the quasi-static approximation to a frequency dependent parameter, an ABC using this α provides good performance through the frequency range that is typically of interest.

III. Numerical Results

The first geometry studied is a uniform microstrip with a conductor width, W, of 2.54 mm, a dielectric thickness, H, of 2.54 mm, and a substrate relative permittivity, ε_r, of 10.2. The inset in Fig. 1 shows the microstrip cross-section and the location and nature of the boundary conditions on the side walls. We use a symmetry boundary condition (PMC) on one side, so that only two of the side walls require ABC's. Fig. 1 compares the voltage errors caused by the second-order Mur ABC [2] and the evanescent ABC of this paper. These errors are found by simulating the microstrip with a computational domain cross-section of 24 × 24 cells and comparing the resulting voltage waveforms with those found when the computational domain cross-section is very large, in this case 110 x 70 cells. Fig. 1 shows that the evanescent ABC provides better accuracy than a second-order Mur boundary condition for this problem.

The second numerical example involves the calculation of the scattering parameters of a microstrip gap from FDTD data [3]. The microstrip conductor width is 0.6 mm, the dielectric thickness is 0.6 mm, and the relative permittivity of the substrate is 9.6. There is a 0.3 mm gap in the microstrip conductor halfway down the line, and the computational domain cross-section is 24 × 24 cells. Fig. 3 compares the errors in the magnitudes of S_{11} and S_{21} introduced by the evanescent and second-order Mur boundary conditions. These errors are computed via comparison with a simulation run with a computational domain cross-section of 110 × 60 cells. Once again, the ABC of this letter provides better results than the second-order Mur condition. Since radiated fields are significant for discontinuity problems of this type, it should be possible to obtain further improvements in accuracy by adding additional traveling wave factors to (1) to create higher-order evanescent boundary conditions.

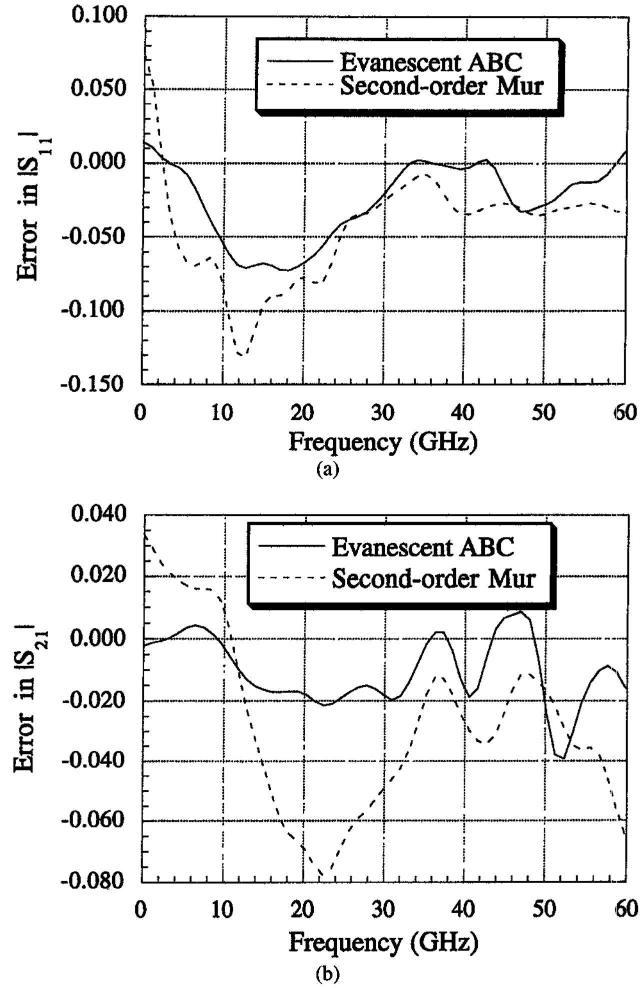

Fig. 2. Error in the magnitude of (a) S_{11} and (b) S_{21} of a microstrip gap introduced by different ABC's. Evanescent boundary uses α at $M\Delta x = 1524$ and α at $N\Delta y = 1257$. Computational domain cross-section is $24\Delta x$ by $24\Delta y$ with Δx and Δy both equal to 0.06 mm.

IV. Conclusion

This communication has presented a boundary condition for FDTD mesh truncation that absorbs both traveling and evanescent waves. The values of the boundary condition parameters are found via the finite-difference solution of a two-dimensional Laplace equation, which requires little additional computation relative to the solution of the associated FDTD problem. When applied to the side walls of a microstrip simulation, the ABC of this letter provides better accuracy than a second-order Mur ABC, and, if the microstrip is uniform, should provide better accuracy than any ABC which cannot model evanescent wave behavior properly.

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